Notes 5.1 Solving Systems of Linear Equations by graphing

Key Idea

Solving a System of Linear Equations by Graphing

Step 1: Graph each equation in the same coordinate plane.

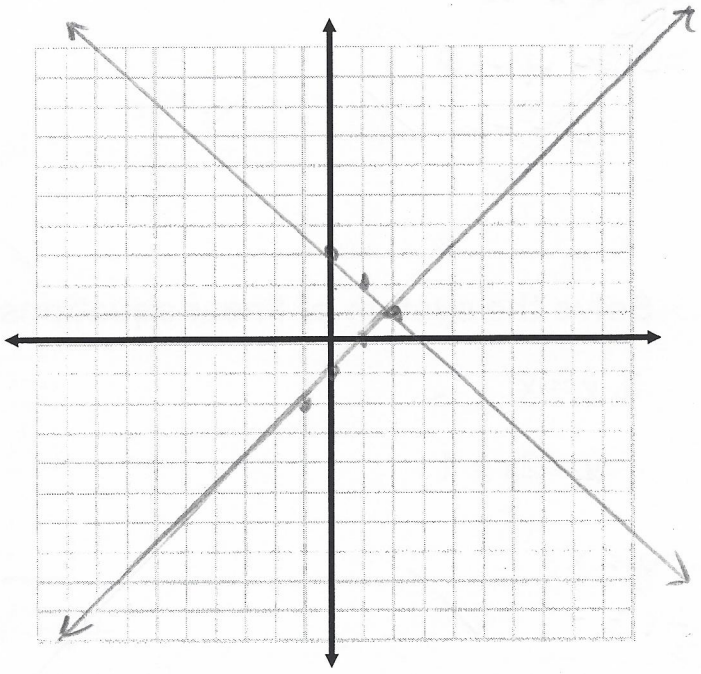
Step 2: Estimate the point of intersection.

Step 3: Check the point from Step 2 by substituting for x and y in each equation of the original system.

Solve the system of linear equations by graphing.

1. $y = x - 1$

   $y = -x + 3$



(2, 1)

$y = x - 1$

$1 = 2 - 1$

$1 = 1$ ✓

$y = -x + 3$

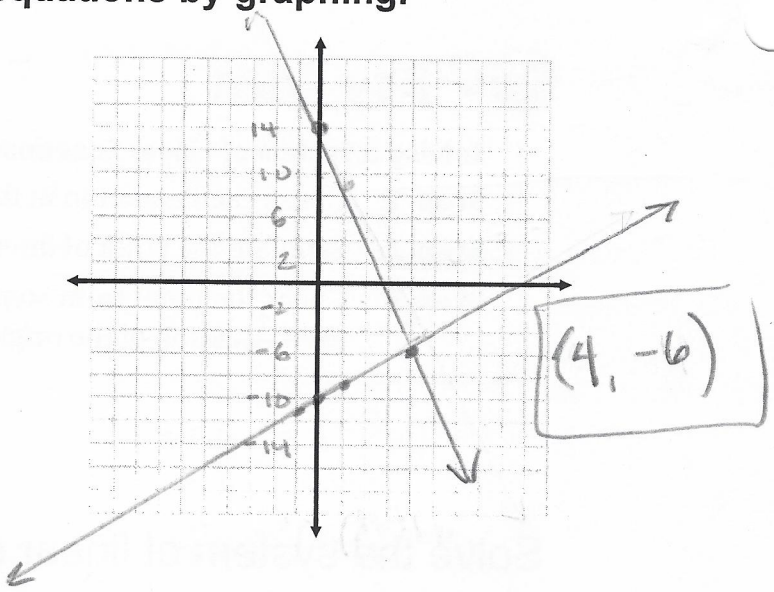
$1 = -2 + 3$

$1 = 1$ ✓

Solve the system of linear equations by graphing.

2. $y = -5x + 14$

$y = x - 10$



$y = -5x + 14$

$-6 = -5(4) + 14$

$-6 = -20 + 14$

$-6 = -6$ ✓

$y = x - 10$

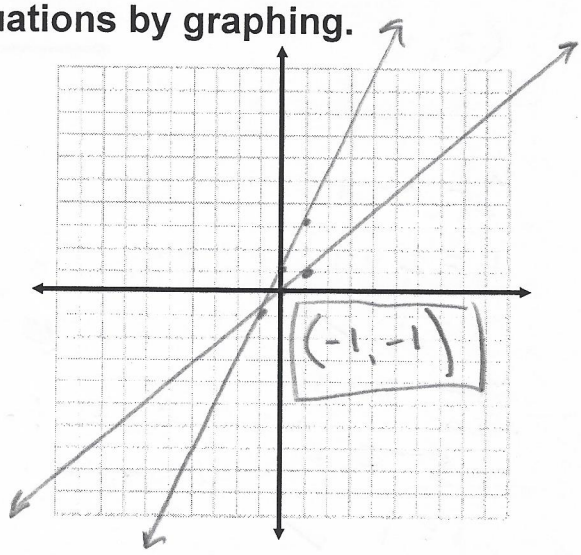
$-6 = 4 - 10$

$-6 = -6$ ✓

Solve the system of linear equations by graphing.

3. $y = x$

$y = 2x + 1$



$y = x$

$-1 = -1$ ✓

$y = 2x + 1$

$-1 = 2(-1) + 1$

$-1 = -1$ ✓

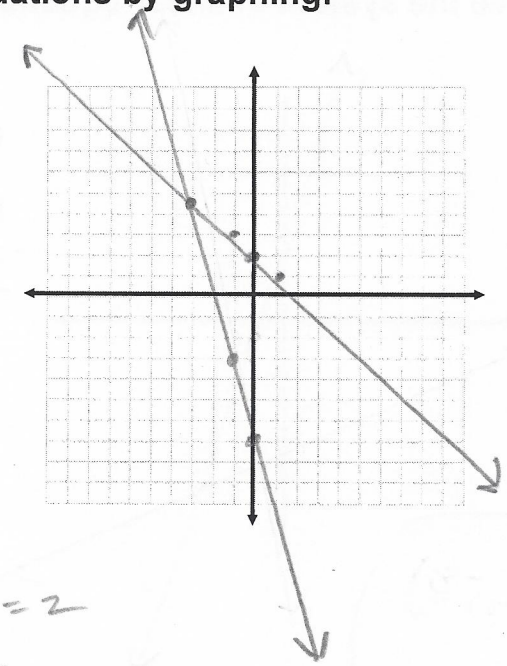
Solve the system of linear equations by graphing.

4. $y = -4x - 7$

$x + y = 2$

$-x \quad -x$

$y = -x + 2$



$(-3, 5)$

$y = -4x - 7$

$5 = -4(-3) - 7$

$5 = 12 - 7$

$5 = 5$ ✓

$x + y = 2$

$-3 + 5 = 2$

$2 = 2$ ✓

Solve the system of linear equations by graphing.

5. $x - y = 5$

$-3x + y = -1$

$x - y = 5$

$-x \quad -x$

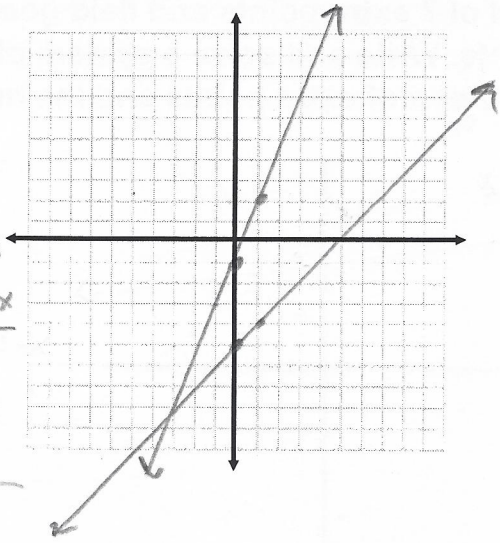
$\frac{-y}{-1} = \frac{-x + 5}{-1}$

$y = x - 5$

$-3x + y = -1$

$+3x \quad +3x$

$y = 3x - 1$



$(-2, -7)$

$x - y = 5$

$-2 - (-7) = 5$

$-2 + 7 = 5$

$5 = 5$ ✓

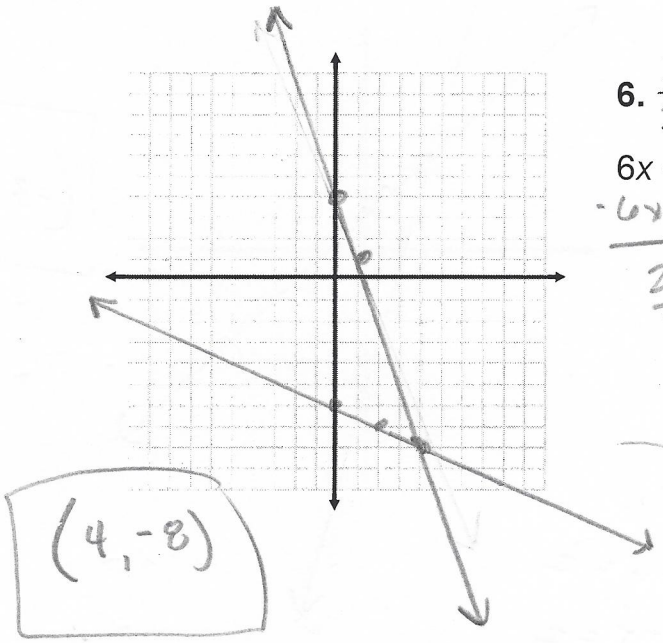
$-3x + y = -1$

$-3(-2) + (-7) = -1$

$6 + (-7) = -1$

$-1 = -1$ ✓

Solve the system of linear equations by graphing.



6. $\frac{1}{2}x + y = -6$

$6x + 2y = 8$

$-6x \quad -6x$

$\frac{2y}{2} = \frac{-6x + 8}{2}$

$y = -3x + 4$

$\frac{1}{2}x + y = -6$

$-\frac{1}{2}x \quad -\frac{1}{2}x$

$y = -\frac{1}{2}x - 6$

$(4, -8)$

$\frac{1}{2}x + y = -6$

$\frac{1}{2}(4) + (-8) = -6$

$2 + -8 = -6$

$-6 = -6$ ✓

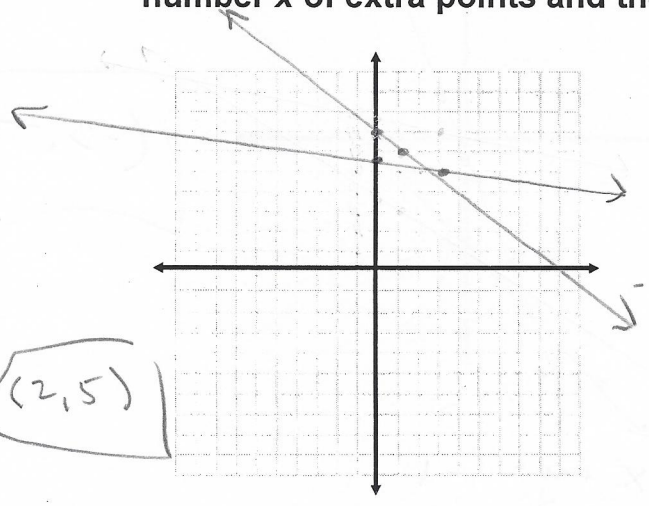
$6x + 2y = 8$

$6(4) + 2(-8) = 8$

$24 + (-16) = 8$

$8 = 8$ ✓

A kicker on a football team scores 1 point for making an extra point band 3 points for making a field goal. The kicker makes a total of 7 extra points and field goals in a game and scores 17 points. Write and solve a system of linear equations to find the number *x* of extra points and the number *y* of field goals.



$1x + 3y = 17$

$x + y = 7$

$x + 3y = 17$

$-x \quad -x$

$\frac{3y}{3} = \frac{-x + 17}{3}$

$y = -\frac{1}{3}x + 5\frac{2}{3}$

$x + y = 7$

$-x \quad -x$

$y = -x + 7$

$(2, 5)$

$x + 3y = 17$

$2 + 3(5) = 17$

$2 + 15 = 17$

$17 = 17$ ✓

$x + y = 7$

$2 + 5 = 7$

$7 = 7$ ✓

Key Idea

Solving a System of Linear Equations by Substitution

Step 1: Solve one of the equations for one of the variables.

Step 2: Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3: Substitute the value from Step 2 into one of the original equations and solve.

Solve the system of linear equations by substitution. Check your solution.

1. $y = 2x + 3$

$y = 5x$

(1, 5)

$5x = 2x + 3$

$-2x \quad -2x$

$3x = 3$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

$y = 5(1)$

$y = 5$

$y = 2x + 3$

$5 = 2(1) + 3$

$5 = 5$ ✓

$y = 5x$

$5 = 5(1)$

$5 = 5$ ✓

Solve the system of linear equations by substitution. Check your solution.

2. $4x + 2y = 0$

$y = \frac{1}{2}x - 5$

(2, -4)

$4x + 2\left(\frac{1}{2}x - 5\right) = 0$

$4x + x - 10 = 0$

$5x - 10 = 0$

$+10 \quad +10$

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

$y = \frac{1}{2}(2) - 5$

$y = 1 - 5$

$y = -4$

$4x + 2y = 0$

$4(2) + 2(-4) = 0$

$8 - 8 = 0$

$0 = 0$ ✓

$y = \frac{1}{2}x - 5$

$-4 = \frac{1}{2}(2) - 5$

$-4 = 1 - 5$

$-4 = -4$ ✓

Solve the system of linear equations by substitution. Check your solution.

3. $x = 5y + 3$

$2x + 4y = -1$

$2(5y+3)+4y=-1$

$10y+6+4y=-1$

$14y+6=-1$

$-6 \quad -6$

$\frac{14y}{14} = \frac{-7}{14}$

$y = -\frac{1}{2}$

$x=5\left(-\frac{1}{2}\right)+3$

$x=-\frac{5}{2}+\frac{3}{1}$

$x=-\frac{5}{2}+\frac{6}{2}$

$x=\frac{1}{2}$

$\left(\frac{1}{2}, -\frac{1}{2}\right)$

$x = 5y + 3$

$\frac{1}{2} = 5\left(-\frac{1}{2}\right)+3$

$\frac{1}{2} = -\frac{5}{2}+\frac{6}{2}$

$\frac{1}{2} = \frac{1}{2}$ ✓

$2x+4y=-1$

$2\left(\frac{1}{2}\right)+4\left(-\frac{1}{2}\right)=-1$

$1+(-2)=-1$

$-1=-1$ ✓

4. You sell lemonade for $2 per cup and orange juice for $3 per cup. You sell a total of 100 cups for $240. Write and solve a system of linear equations to find the number of cups of lemonade and the number of cups of orange juice you sold.

x = lemonade cups

y = orange juice cups

60 cups of lemonade

40 cups of orange juice

$\begin{cases} 2x+3y=240 \\ x+y=100 \end{cases}$

$x+y=100$

$-y \quad -y$

$x=-y+100$

$2(-y+100)+3y=240$

$-2y+200+3y=240$

$y+200=240$

$-200 \quad -200$

$y=40$

$x+40=100$

$-40 \quad -40$

$x=60$

$2x+3y=240$

$2(60)+3(40)=240$

$120+120=240$

$240=240$ ✓

$x+y=100$

$60+40=100$

$100=100$ ✓

Notes 5.3 Solving Systems of Linear Equations by Elimination

Key Idea

Solving a System of Linear Equations by Elimination

Step 1: Multiply, if necessary, one or both equations by a constant so at least 1 pair of like terms has the same or opposite coefficients.

Step 2: Add or subtract the equations to eliminate one of the variables.

Step 3: Solve the resulting equation for the remaining variable.

Step 4: Substitute the value from Step 3 into one of the original equations and solve.

Solve the system of linear equations by elimination. Check your solution.

1. $2x - y = 9$
$+ \quad 4x + y = 21$

$\frac{6x}{6} = \frac{30}{6}$

$x = 5$

$4(5) + y = 21$

$20 + y = 21$

$-20 \quad -20$

$y = 1$

$(5, 1)$

$2x - y = 9$

$2(5) - 1 = 9$

$10 - 1 = 9$

$9 = 9$ ✓

$4x + y = 21$

$4(5) + 1 = 21$

$20 + 1 = 21$

$21 = 21$ ✓

Solve the system of linear equations by elimination. Check your solution.

2. $-5x + 2y = 13$

$+ \quad 5x + y = -1$

$\frac{3y}{3} = \frac{12}{3}$

$y = 4$

$5x + 4 = -1$

$-4 \quad -4$

$\frac{5x}{5} = \frac{-5}{5}$

$x = -1$

$(-1, 4)$

$-5x + 2y = 13$

$-5(-1) + 2(4) = 13$

$5 + 8 = 13$

$13 = 13$ ✓

$5x + y = -1$

$5(-1) + 4 = -1$

$-5 + 4 = -1$

$-1 = -1$ ✓

Solve the system of linear equations by elimination. Check your solution.

3. $3x + 4y = -6$ ← multiply by -1 so you can eliminate by adding.

$7x + 4y = -14$

$(+) \; -3x - 4y = 6$

$\frac{4x}{4} = \frac{-8}{4}$

$x = -2$

$3(-2) + 4y = -6$

$-6 + 4y = -6$

$+6 \quad +6$

$\frac{4y}{4} = \frac{0}{4}$

$y = 0$

$(-2, 0)$

$3x + 4y = -6$

$3(-2) + 4(0) = -6$

$-6 = -6$ ✓

$7x + 4y = -14$

$7(-2) + 4(0) = -14$

$-14 = -14$ ✓

Notes 5.3 Solving Systems of Linear Equations by Elimination

Key Idea

Solving a System of Linear Equations by Elimination

Step 1: Multiply, if necessary, one or both equations by a constant so at least 1 pair of like terms has the same or opposite coefficients.

Step 2: Add or subtract the equations to eliminate one of the variables.

Step 3: Solve the resulting equation for the remaining variable.

Step 4: Substitute the value from Step 3 into one of the original equations and solve.

Solve the system of linear equations by elimination. Check your solution.

4. $3x + y = 11$ ← multiply by -2 so you can eliminate by addition
   $6x + 3y = 24$

$(+)\ -6x - 2y = -22$

$y = 2$

$3x + 2 = 11$
$-2 \quad -2$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

$3x + y = 11$
$3(3) + 2 = 11$
$9 + 2 = 11$
$11 = 11$ ✓

$6(3) + 3(2) = 24$
$18 + 6 = 24$
$24 = 24$ ✓

$(3, 2)$

Solve the system of linear equations by elimination. Check your solution.

5. $4x - 5y = -19$

$-x - 2y = 8$ ← multiply by 4

$4x - 5y = -19$

$(+)\ -4x - 8y = 32$

$-13y = 13$

$\frac{-13y}{-13} = \frac{13}{-13}$

$y = -1$

$-x - 2(-1) = 8$

$-x + 2 = 8$

$-2 \quad -2$

$-x = 6$

$\frac{-x}{-1} = \frac{6}{-1}$

$x = -6$

(-6, -1)

$4x + 5y = -19$

$4(-6) - 5(-1) = -19$

$-24 + 5 = -19$

$-19 = -19$ ✓

$-x - 2y = 8$

$-(-6) - 2(-1) = 8$

$6 + 2 = 8$

$8 = 8$ ✓

Solve the system of linear equations by elimination. Check your solution.

6. $5y = 15 - 5x$

$y = -2x + 3$ ← multiply by -5

$5y = -5x + 15$

$(+)\ -5y = 10x - 15$

$\frac{0}{5} = \frac{5x}{5}$

$x = 0$

$y = -2(0) + 3$

$y = 3$

(0, 3)

$5y = 15 - 5x$

$5(3) = 15 - 5(0)$

$15 = 15$ ✓

$y = -2x + 3$

$3 = -2(0) + 3$

$3 = 3$ ✓

7. A landscaper buys 4 peonies and 9 geraniums for $190. Another landscaper buys 5 peonies and 6 geraniums for $185. Write and solve a system of linear equations to find the cost of each peony.

p = peony cost

g = geranium cost

$$\begin{cases} 4p + 9g = 190 \\ 5p + 6g = 185 \end{cases}$$

$5(4p + 9g = 190) \rightarrow 20p + 45g = 950$

$-4(5p + 6g = 185) \rightarrow -20p - 24g = -740$

$\frac{21g}{21} = \frac{210}{21}$

$g = 10$

$4p + 9(10) = 190$

$4p + 90 = 190$

$-90 \quad -90$

$\frac{4p}{4} = \frac{100}{4}$

$p = 25$

The peony cost $25

$4p + 9g = 190$

$4(25) + 9(10) = 190$

$100 + 90 = 190$

$190 = 190$ ✓

$5p + 6g = 185$

$5(25) + 6(10) = 185$

$125 + 60 = 185$

$185 = 185$ ✓

Notes 5.4 Solving Special Systems of Equations

Summary

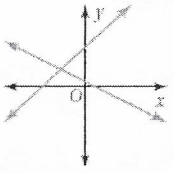
Methods for Solving Systems of Linear Equations

Method	When to Use
Graphing *(Lesson 5.1)*	To estimate solutions
Substitution *(Lesson 5.2)*	When one of the variables in one of the equations has a coefficient of 1 or −1
Elimination *(Lesson 5.3)*	When at least 1 pair of like terms has the same or opposite coefficients
Elimination (Multiply First) *(Lesson 5.3)*	When one of the variables cannot be eliminated by adding or subtracting the equations

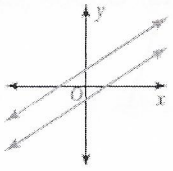
Key Idea

Solutions of Systems of Linear Equations

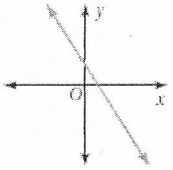
A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.



One solution
The lines intersect.



No solution
The lines are parallel.



Infinitely many solutions
The lines are the same.

Solve the system of linear equations. Check your solution.

1. $y = -x + 3$
   $y = -x + 5$

$-x+5 = -x+3$
$+x \qquad +x$
$5 \neq 3$

no solution

Solve the system of linear equations. Check your solution.

2. $y = -5x - 2$

   $5x + y = 0$

$5x - 5x - 2 = 0$

$0 - 2 = 0$

$-2 = 0$

no solution

Solve the system of linear equations. Check your solution.

3. $x = 2y + 10$

   $2x + 3y = -1$

$(4, -3)$

$2(2y+10) + 3y = -1$

$4y + 20 + 3y = -1$

$7y + 20 = -1$

$-20 \quad -20$

$\frac{7y}{7} = \frac{-21}{7}$

$y = -3$

$x = 2(-3) + 10$

$x = -6 + 10$

$x = 4$

$x = 2y + 10$

$4 = 2(-3) + 10$

$4 = -6 + 10$

$4 = 4$ ✓

$2x + 3y = -1$

$2(4) + 3(-3) = -1$

$8 - 9 = -1$

$-1 = -1$ ✓

Solve the system of linear equations. Check your solution.

4. $x + y = 3$

$+ \quad x - y = -3$

$\frac{2x}{2} = \frac{0}{2}$

$x = 0$

$0 + y = 3$

$y = 3$

$(0, 3)$

$x + y = 3$

$0 + 3 = 3$

$3 = 3$ ✓

$x - y = -3$

$0 - 3 = -3$

$-3 = -3$ ✓

Solve the system of linear equations. Check your solution.

5. $2x + y = 5$ $\quad \cdot -2$

$4x + 2y = 0$

$-4x - 2y = -10$

$0 \neq -10$

no solution

Solve the system of linear equations. Check your solution.

6. $2x - 4y = 10$
$-12x + 24y = -60$

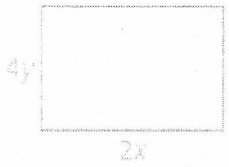
$12x - 24y = 60$
$-12x + 24y = -60$

$0 = 0$

infinitely many solutions

The perimeter of Rectangle A is 54 units. The perimeter of Rectangle B is 108 units. Write and solve a system of linear equations to find the values of *x* and *y*.

Rectangle A

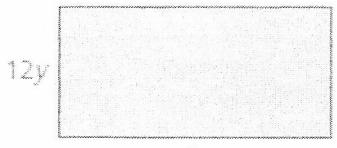


$-3(2x + 4y = 54)$

$6x + 12y = 108$
$-6x - 12y = -162$

$0 = -54$

Rectangle B



no solution